Systems of Equations

System of equations two or more equations where you want to find a sloution that makes all of them true simultaneousley (the same point makes them all true).

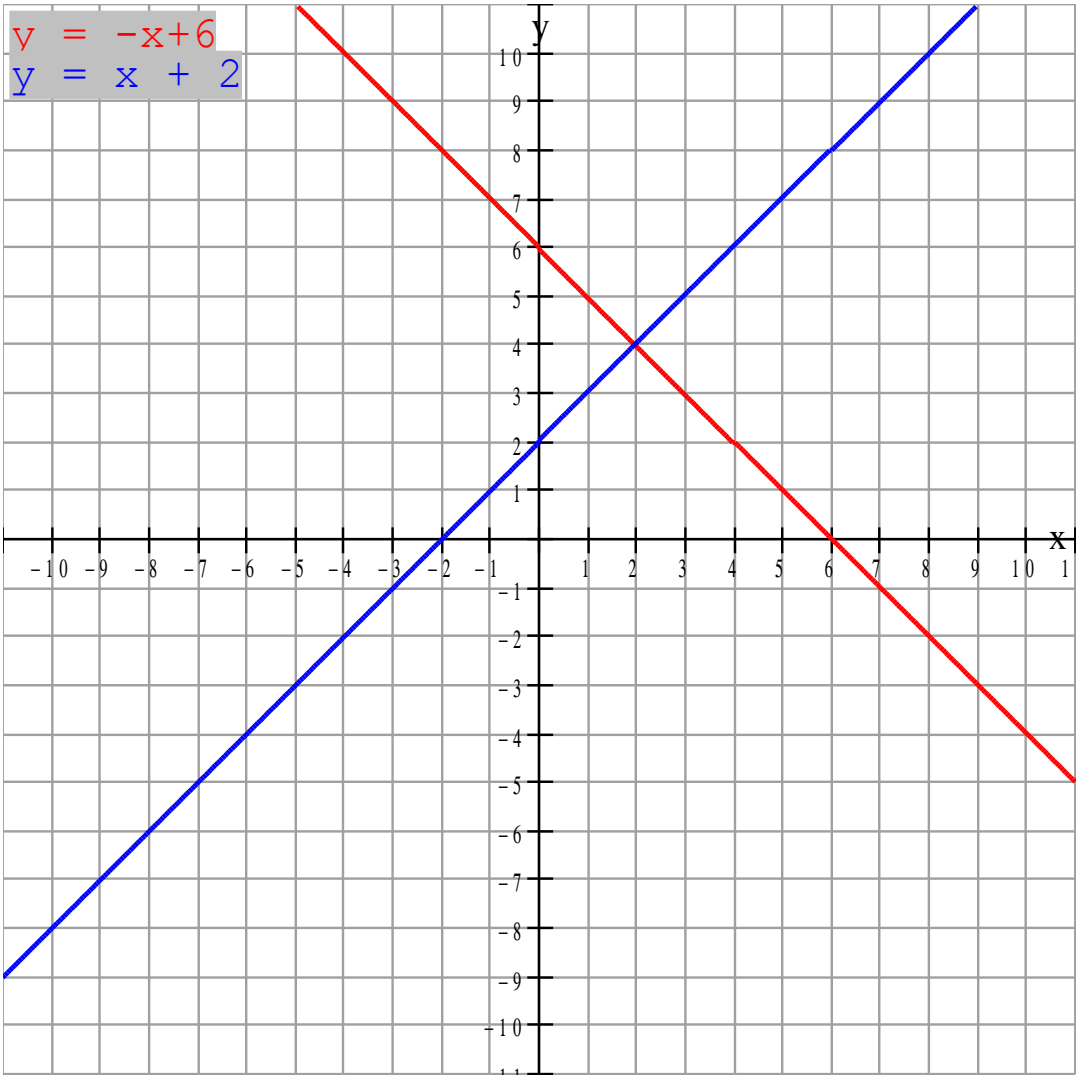
Solution of a system of equations the point that makes all the equations true (satisfies all the equations, the point that all the lines go through).

Three types of systems of equations
Systems are classified as either inconsistent or consistent. Inconsistent means there is no solution to the system. Consistent means there is at least one solution. Consistent systems come in two types, independent which means only one solution, and dependent which means infinitely many solutions.

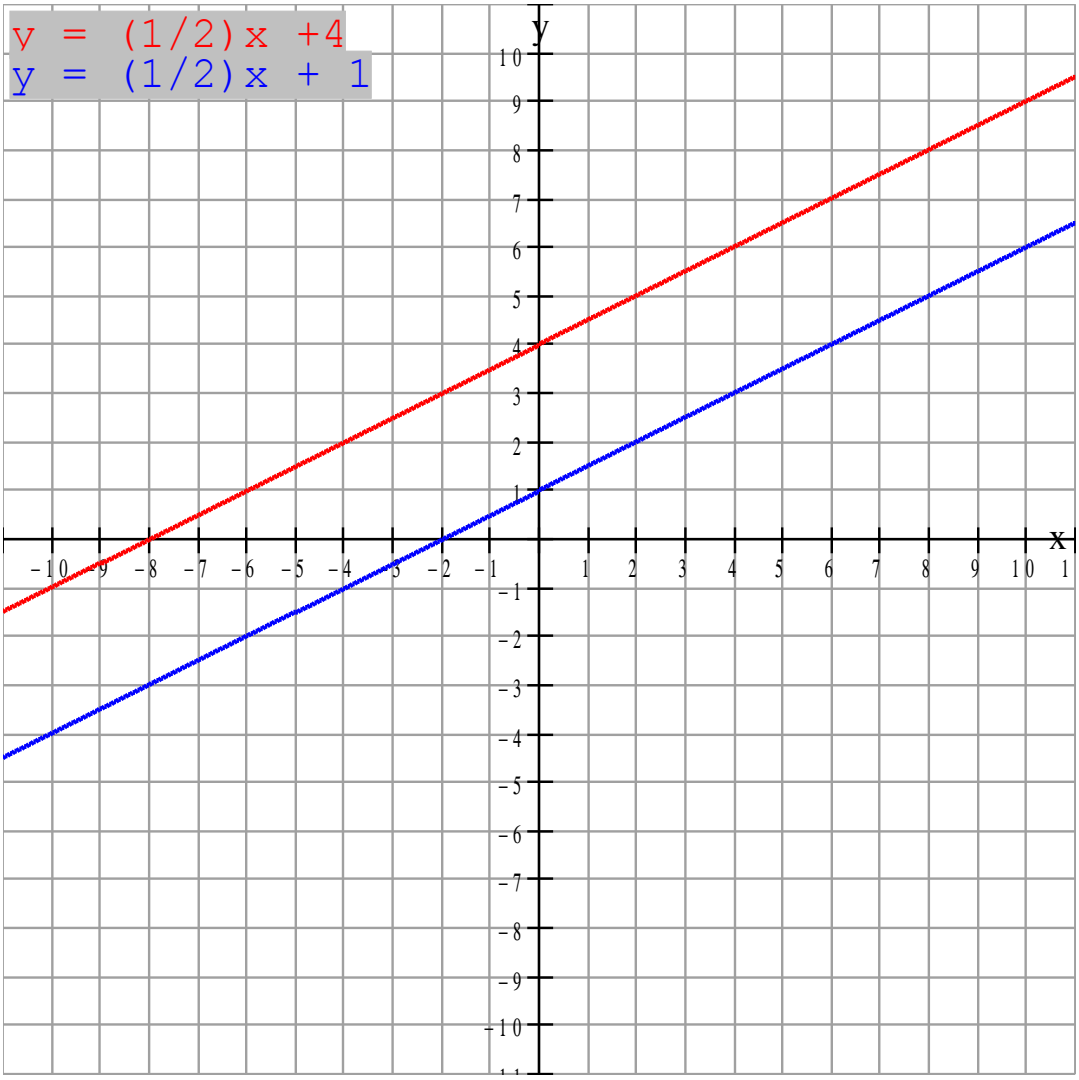
- **No solution** (same slopes, $m_1 = m_2$; different y-intercepts, $b_1 \neq b_2$)
  Lines are parallel they never intersect, there is no point that all the lines pass through, there is no solution. The equations are inconsistent because they do not have a solution.
- **One solution** (different slopes, $m_1 \neq m_2$)
  Lines intersect, pass through one point. The equations are consistent because they have at least one solution. The equations are also independent because they only have one point in common.
- **Infinitely many solutions** (same slope, $m_1 = m_2$; same y-intercepts, $b_1 = b_2$)
  The lines are on top of each other, they cover each other, they are the same line. Every point on one line is also on the other line so there are infinitely many points that satisfy both equations although not every point of the coordinate plane is a solution of the system of equations. The equations are consistent because they have at least one solution. The equations are also dependent because they have infinitely many points in common. The graphs coincide, the lines are coincident.

Parallel lines same slopes, different y-intercepts, lines never intersect. The symbol for parallel lines is $\parallel$.

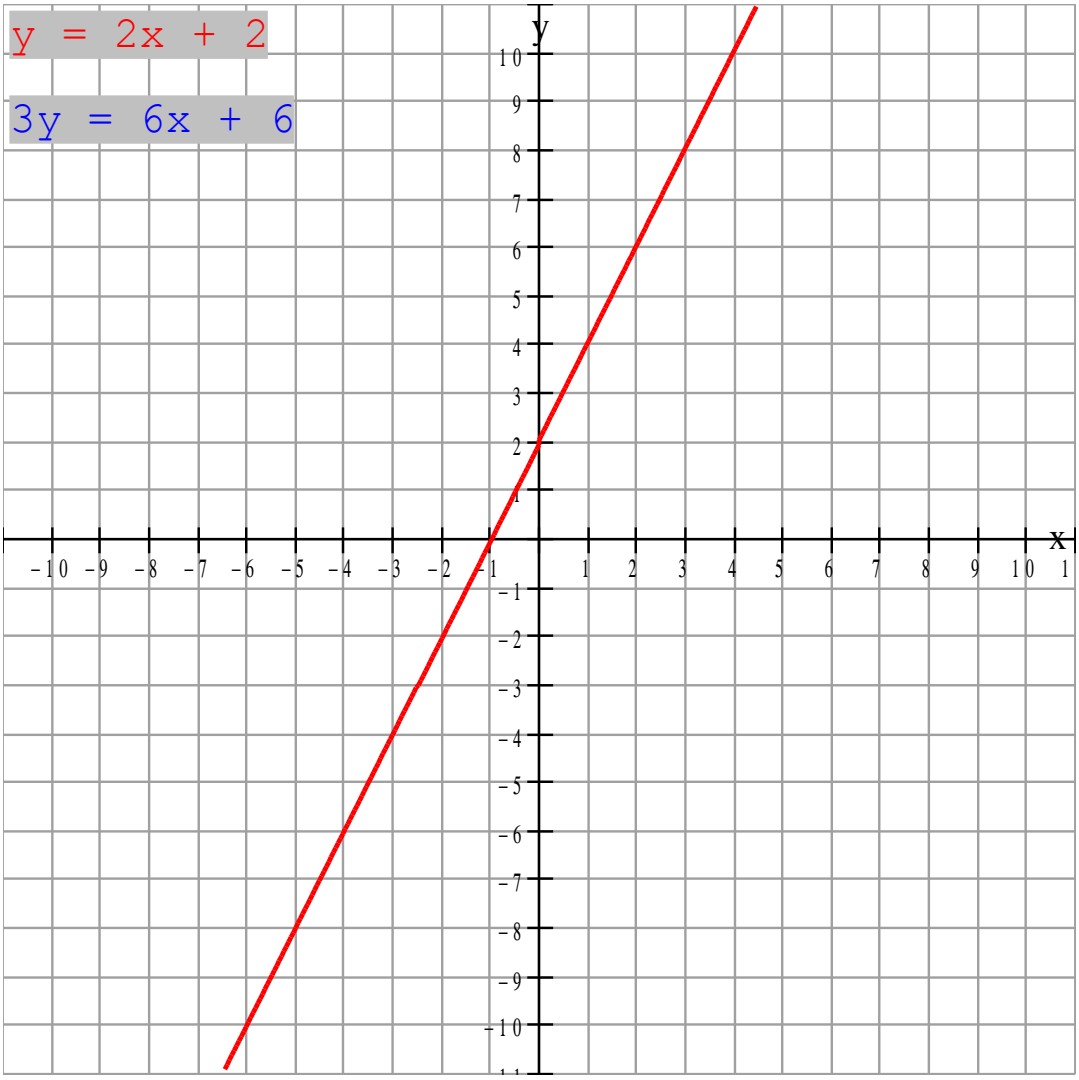
Perpendicular lines slopes are the negative reciprocals of each other. They intersect at a 90º angle (right angle). The symbol for perpendicular lines is $\perp$.



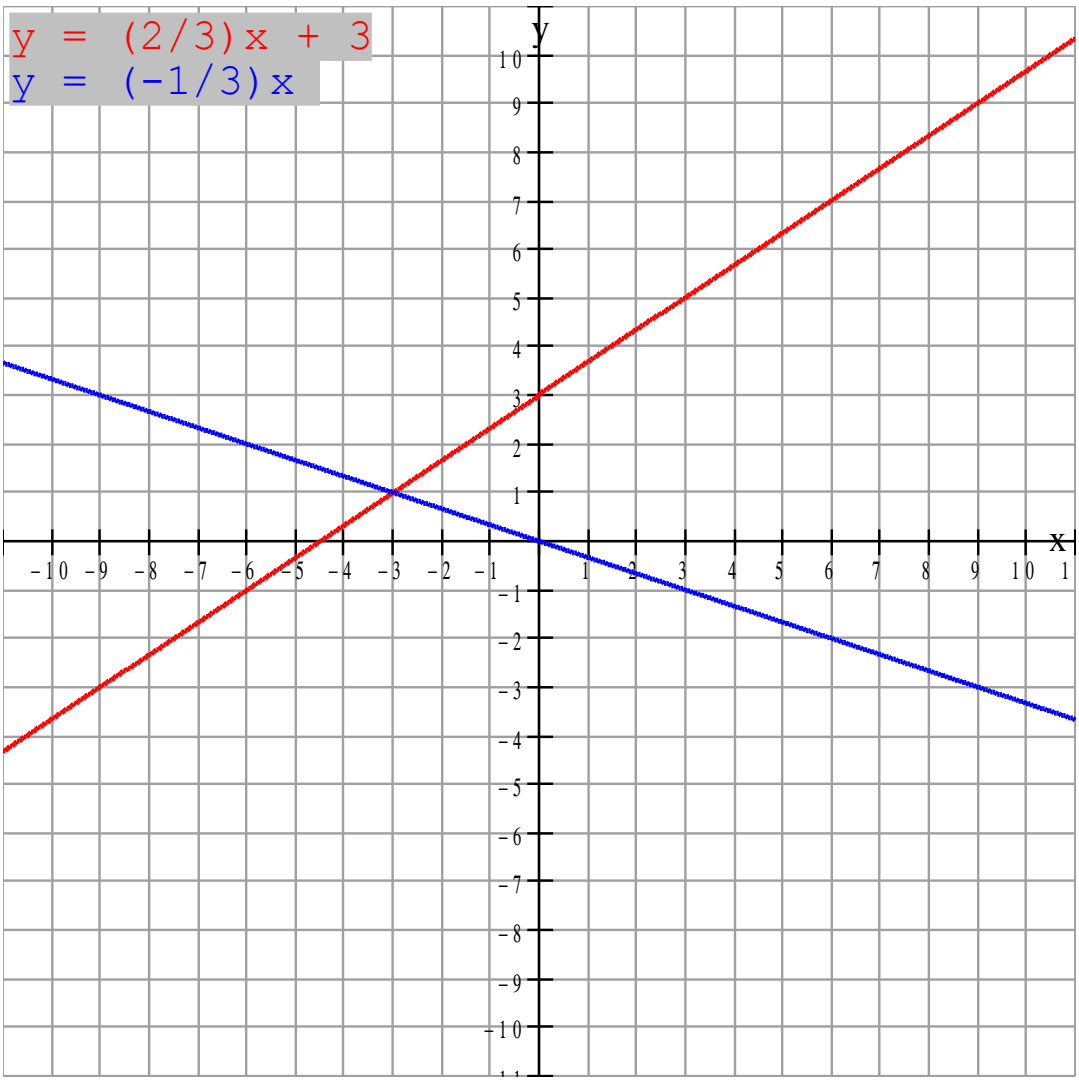
The solution of this system of equations is the point where both lines intersect, (2, 4). These lines are perpendicular because the slope of the red line is -1 and the slope of the blue line is 1. The slope of one line is the negative reciprocal of the slope of the other line. Algebraically this looks like $m_1 = -1/m_2$. This system of equations is consistent and independent.



These lines are parallel, they never intersect. They have the same slopes (slope of the red line is ½ and the slope of the blue line is ½) but different y-intercepts (y-intercept of red line is 4 and the y-intercept of the blue line is 1). There is no solution to this system because there is no point where the two lines intersect. This system of equations is inconsistent.



You only see one line on this graph because the lines are on top of each other. It does not look like they are the same line from the equations but when both equations are solved for y (put in slope-intercept form) it is easy to see they are the same. There are infinitely many solutions to this system because every point on one line is also on the other. Note that even though there are infinitely many points that make both equations true not every point on the coordinate plane is a solution of the system of equations. For example the point (0, 0) is not a solution to the system and neither is (1, 1). This system of equations is consistent, dependent, and coincident.



This is a typical system of equations, there is one point where the two lines intersect (-3, 1), this is the solution of this system of equations. This system of equations is consistent and independent.