Variance and Standard Deviation

I need to define some terms first because they are used in the definitions of variance and standard deviation.

Given a set of data with n elements $\{x_1, x_2, x_3, x_4, \ldots, x_n\}$, **the mean of the set, *M*,** is defined as the arithmetic average of the numbers in the set:

$$M=\frac{x_1+x_2+x_3+x_4+\ldots+x_n}{n}$$

The **deviation** d_i **, of an element** x_i **, of the set,** is the difference between the element and the mean.

$$d_i=x_i-M$$

The **variance of the set, V,** is defined as the arithmetic average of the squares of the deviations (the mean of the squares of the deviations).

$$V=\frac{d_1^2+d_2^2+d_3^2+\ldots+d_n^2}{n} \quad \text{or} \quad V=\frac{(x_1-M)^2+(x_2-M)^2+(x_3-M)^2+\ldots+(x_n-M)^2}{n}$$

The **standard deviation of the set, S,** is defined as the principal (positive) square root of the variance.

$$S=\sqrt{V}$$

Example

Given the set {2, 4, 6, 8, 10}:

The **mean** is found using this formula $M=\frac{x_1+x_2+x_3+x_4+\ldots+x_n}{n}$

So $M=\frac{2+4+6+8+10}{5} = \frac{30}{5}$ = 6, so M = 6.

The **deviations** of each element in the set are found using $d_i=x_i-M$

So $d_1=2-6=-4$, $d_2=4-6=-2$, $d_3=6-6=0$, $d_4=8-6=2$, $d_5=10-6=4$.

The **variance** is found using

$$V=\frac{d_1^2+d_2^2+d_3^2+\ldots+d_n^2}{n} \quad \text{or} \quad V=\frac{(x_1-M)^2+(x_2-M)^2+(x_3-M)^2+\ldots+(x_n-M)^2}{n}$$

So $V=\frac{(-4)^2+(-2)^2+0^2+2^2+4^2}{5} = \frac{16+4+0+4+16}{5} = \frac{40}{5}$ = 8.

The **standard deviation** is found using $S=\sqrt{V}$ So $S=\sqrt{8}=2\sqrt{2}$.