Formulas for Arc Length

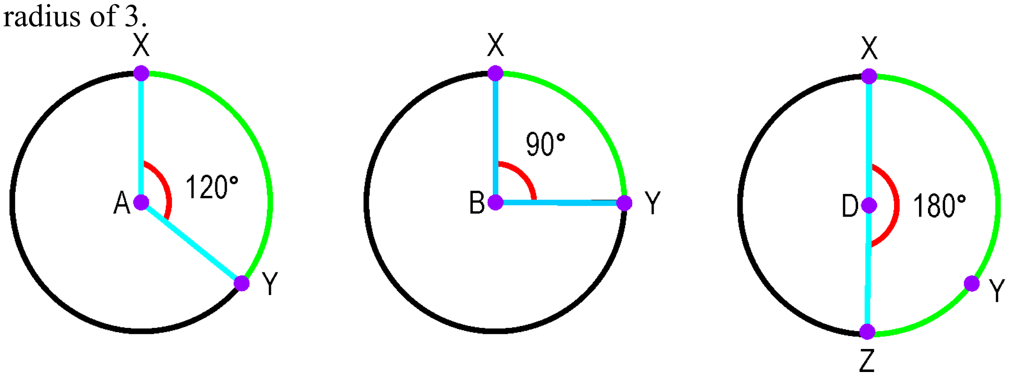
It helps to think of a pizza when working with these types of problems. *The arc length is the length of the crust on the piece of pizza you get.* If you get the whole pizza then the arc length is the circumference of the pizza. If you get half the pizza then the arc length is half the circumference of the pizza.

In each of the formulas let C stand for circumference, r stand for radius, d stand for diameter, and l stand for arc length. Use the formula $C = 2\pi r$ to calculate the circumference of a circle when the radius is given. Use the formula $C = \pi d$ to calculate the circumference of a circle when the diameter is given.

You can use $\frac{C}{360°} = \frac{l}{\text{measure of the central angle}}$ or $\frac{\text{measure of the central angle}}{360°} = \frac{l}{C}$

Example 1, finding the arc length.

What is the length of arc XY in circle A and B? What is the length of arc XYZ in Circle D? Each circle has a radius of 3.



Begin by writing the formula.

$$\frac{C}{360°} = \frac{l}{\text{measure of the central angle}} \quad \text{Or} \quad \frac{\text{measure of the central angle}}{360°} = \frac{l}{C}$$

Solving either of the formulas for l you get $l = \frac{\text{measure of the central angle}}{360°} \cdot C$

For circle A you get $l = \frac{120°}{360°} \cdot C$ which simplifies to $l = \frac{1}{3} \cdot C$ and since $C = 6\pi$ you end up with $l = 2\pi$ as the arc length for arc XY.

For circle B you get $l = \frac{90°}{360°} \cdot C$ which simplifies to $l = \frac{1}{4} \cdot C$ and since $C = 6\pi$ you end up with $l = \frac{3}{2}\pi$ as the arc length for arc XY.

For circle D you get $l = \frac{180°}{360°} \cdot C$ which simplifies to $l = \frac{1}{2} \cdot C$ and since $C = 6\pi$ you end up with $l = 3\pi$ as the arc length for arc XYZ.

Formulas for Area of a Sector

It helps to think of a pizza when working with these type of problems. *The area of the sector is the size of the piece of pizza you get.* If you get the whole pizza then the area of the sector is the area of the pizza. If you get half the pizza then the area of the sector is half the area of the pizza.

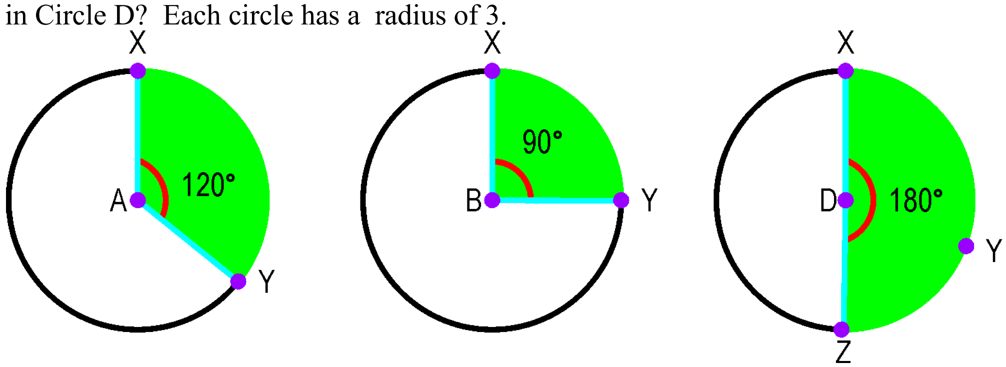
In each of the formulas let A stand for the area of the circle, r stand for radius, d stand for diameter, and $Asec$ stand for the area of the sector. Use the formula $d = 2r$ to calculate the radius when the diameter is given.

You can use either of the formulas below:

$$\frac{\text{area of circle}}{360°} = \frac{\text{area of sector}}{\text{measure of central angle}} \quad \text{or} \quad \frac{\text{measure of central angle}}{360°} = \frac{\text{area of sector}}{\text{area of circle}}$$

Example 2, finding the area of a sector.

What is the area of sector XAY in circle A? What is the area of sector XBY in circle B? What is the area of sector XDZY in Circle D? Each circle has a radius of 3.



Begin by writing the formula.

$$\frac{A}{360°} = \frac{Asec}{\text{measure of the central angle}} \quad \text{Or} \quad \frac{\text{measure of the central angle}}{360°} = \frac{Asec}{A}$$

Solving either of the formulas for $Asec$ you get $Asec = \frac{\text{measure of the central angle}}{360°} \bullet A$

For circle A you get $Asec = \frac{120°}{360°} \bullet A$ which simplifies to $Asec = \frac{1}{3} \bullet A$ and since $A = 9\pi$ you end up with $Asec = 3\pi$ as the area for sector XAY.

For circle B you get $Asec = \frac{90°}{360°} \bullet A$ which simplifies to $Asec = \frac{1}{4} \bullet A$ and since $A = 9\pi$ you end up with $Asec = \frac{9}{4}\pi$ as the area for sector XBY.

For circle D you get $Asec = \frac{180°}{360°} \bullet A$ which simplifies to $Asec = \frac{1}{2} \bullet A$ and since $A = 9\pi$ you end up with $Asec = \frac{9}{2}\pi$ as the area for sector XDZY.

How Arc Length and Area of a Sector are Related

$$\frac{arc\ length}{circumference\ of\ circle} = \frac{area\ of\ sector}{area\ of\ circle}$$