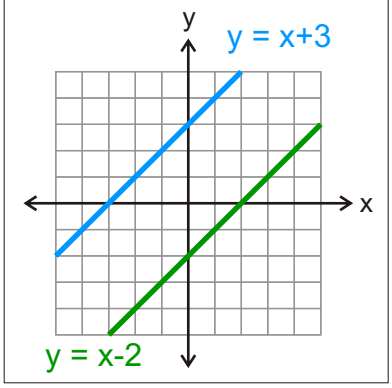
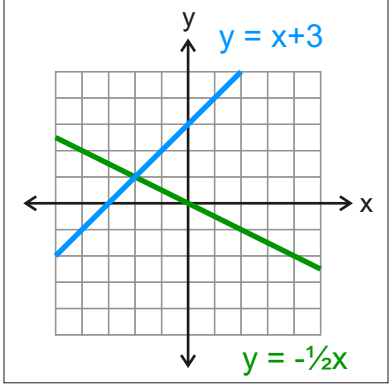
Classifying Systems of Linear Equations

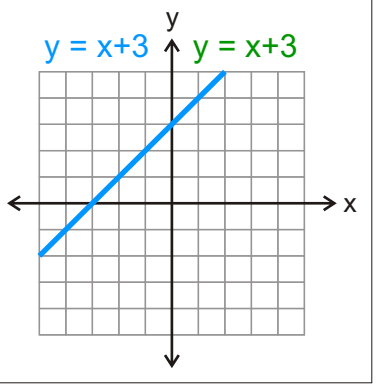
CASE 1
Lines are parallel. There is no solution to the system of equations.



CASE 2
Lines intersect in one point. One solution to the system of equations.



CASE 3
The lines coincide. Infinitely many solutions to the system of equations.



Systems of equations are *inconsistent* if they do not have a solution. If they have at least one solution they are *consistent* (because they consist of at least one solution).

Systems of equations fall into three categories:

- *Inconsistent* – the systems that do not have a solution.
- *Consistent and independent* – the systems that have exactly one solution.
- *Consistent and dependent* – the systems that have infinitely many solutions.

Case 1: Lines are parallel. There is no solution.

The lines do not intersect at any point so there is no solution. A system with no solution is inconsistent.
The system is inconsistent.

$y = x - 2 \quad y = m_1x + b_1 \quad m_1 = 1 \quad b_1 = -2$
$y = x + 3 \quad y = m_2x + b_2 \quad m_2 = 1 \quad b_2 = 3$

The slopes are always the same (m_1 and m_2 are equal).
The y-intercepts are always different (b_1 and b_2 are different).

Case 2: Lines intersect in one point. There is exactly one solution.

The solution is the point where the lines intersect. This system is consistent (consists of a solution) because it has at least one solution. The system is independent because it is consistent and it has exactly one solution.
The system is consistent and independent.

$y = -\frac{1}{2}x \quad y = m_1x + b_1 \quad m_1 = -1/2 \quad b_1 = 0$
$y = x + 3 \quad y = m_2x + b_2 \quad m_2 = 1 \quad b_2 = 3$

The slopes will always be different (m_1 and m_2 are different).
The y-intercepts may or may not be different. (b_1 and b_2 may or may not be different.)

Case 3: The lines are on top of each other. There are infinitely many solutions.

Every point on one line is also on the other line under it. This makes every point on the line a solution. The blue line is covering the green line. The lines coincide, *they are coincident*. The system is consistent because it consists of at least one solution. The system is dependent because there are infinitely many solutions.
The system is consistent and dependent.

$y = x + 3 \quad y = m_1x + b_1 \quad m_1 = 1 \quad b_1 = 3$
$y = x + 3 \quad y = m_2x + b_2 \quad m_2 = 1 \quad b_2 = 3$

The slopes are always the same (m_1 and m_2 are equal).
The y-intercepts are always the same (b_1 and b_2 are equal).